New results of the analysis of the Alcala de Ebro Village (Zaragoza, Spain) gravity anomaly with a set of point sources

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Introduction

Several terrain's collapses took place in the last years inside Alcala de Ebro village (Zaragoza, Spain). Ebro River is close to the village and acts on this zone in an active way. The existence of cavities filled with water or sediments is supposed. The depth of these cavities may be from 12 to 20 m. Besides, there are no other geological studies in the area that provide more information. That is why we tried to use the available geophysical information for this purpose. The Alcala de Ebro (Zaragoza) gravity anomaly was studied with a set of point masses model (Zidarov, 1968; Zidarov, Zhelev, 1970; Zhelev, 1972; Zidarov, 1990; Zhelev et al., 1996) — one entirely Bulgarian method, elaborated in our Department after the ideas and under the scientific guidance of the late D-r D. P. Zidarov.

Mathematical formulation of the problem

The solution of the given problem by the above mentioned method (Zidarov, 1968, 1990) reduces mainly to the solution of the following non linear system of equations $f(x) = y$, where $x(x_1, x_2, ..., x_n)$ is the vector of the unknown parameters (co-ordinates — ξ_k, η_k, ζ_k and masses m_k, $k = 1, ..., n/4$ of the point sources (PS)), which must be determined on the basis of the vector of the observations $y^*(y_1, y_2, ..., y_N)$, while $f^*(f_1, f_2, ..., f_N)$ is a vector of non linear functions (the symbol * means transposition). In this case the functions f_i, $i = 1, ..., N$, can be defined by the analytical expression for the gravity effect

$$f_i(x) = \sum_{k=1}^{n/4} \frac{\gamma m_k (z_i - \varsigma_k)}{R_{ik}^{3/2}}, \quad R_{ik}^2 = (X_i - \xi_k)^2 + (Y_i - \eta_k)^2 + (Z_i - \zeta_k)^2,$$

($\gamma = 66.7 \cdot 10^{-12}$ m^3 kg^{-1} s^{-2} is the gravity constant) of a set of n/4 ES in N points of observation with co-ordinates — X_i, Y_i, Z_i, $i = 1, ..., N$, located over the Earth's surface.

For the representation of the trend of the field, when necessary, a part $a + bX + cY + ...$ (X and Y are the co-ordinates of the observational points) of a polynomial is used, whose coefficients — a, b, c, ... are determined in the process of optimisation, together with the rest of the unknowns (Zhelev, 1991, 1994). Alternatively, more PS can be included in the model for this purpose (Zhelev, 1991, 1994). Their parameters can be specified in the same way. Usually, in order to represent the trend, they must lie significantly deeper than the rest. Of course, the best thing to do here is to try to remove the trend before further interpretation, but this is not always possible with the needed precision. Even so it must be tried, because in all cases, this can considerably ease the optimisation in the next step.

Numerical results

As was already mentioned, the above described gravity anomaly was treated with this method (Zidarov, 1965, 1968, 1990). An appropriate Computer Program on FORTRAN 77, worked out by Zh. Zhelev (1972), was applied for this purpose. The Bouguer gravity was used obtained on the basis of the surface registrations. The observed anomaly and the corresponding trend were represented with 15 PS and the linear part of a polynomial. The optimisation was carried out after the Marquardt (1963) method. A part of the results obtained — the parameters of the elementary sources — x, y and z co-ordinates and the masses, respectively — ξ_k, η_k, ζ_k, m_k and some quantities connected with the corresponding errors in the solution are presented in the table 1. Besides, the following parameters are listed there for convenience:

Table 1. Solution of the Inverse Gravity Problem with a system of elementary sources — 15 point sources and a linear trend represent the local field of the "Zaragoza" gravity anomaly

F	G	K_v[%]	k	ξ_k[m]	η_k [m]	ζ_k [m]	M_k [kg/1.5.10^2]
0.28 . 10^{10}	0.64 . 10^8	557.75	1	893.00	1039.00	-158.00	-2109.00
			2	1109.00	1192.00	-198.00	868.00
			3	1304.00	843.00	-182.00	45639.00
			4	961.00	1130.00	-210.00	91.00
			5	920.00	1162.00	-164.00	-6089.00
			6	1160.00	1249.00	85.00	-821350.00
			7	1171.00	1186.00	87.00	1516400.00
Initial approximations:			8	991.00	1250.00	-160.00	36380.00
			9	1007.00	1240.00	-158.00	-737570.00
			10	1173.00	1119.00	-98.00	-6954100.00
			11	835.00	1228.00	-91.55	-30119.00
			12	1174.00	1112.00	-97.91	3246700.00
			13	1008.00	11240.00	-158.00	685310.00
			14	1171.00	1125.00	-101.00	3352200.00
:			15	1209.00	1128.00	-100.00	163690.00
Parameters of the trend			16	-693.54			
0.20 . 10^5	0.18 . 10^2	1.46	1	931.60	1152.65	-143.99	-67000.01
			2	1023.93	1228.80	-184.73	2581.46
			3	1231.34	958.26	-46.31	59336.24
			4	938.43	1145.25	-130.41	111282.49
			5	836.32	1240.76	-86.26	-35613.06
			6	994.50	1121.95	-61.37	988418.81
			7	1016.91	1118.77	-51.25	1354955.70
Solution:			8	991.34	1251.71	-141.32	98347.95
			9	999.52	1246.53	-118.30	-663745.83
N = 306, n = 60, m = 1;			10	1120.60.	1136.24	-70.71	-7049119.20
			11	999.80	1514.64	-202.05	-139901.19
σ_v = 1.70 mgal			12	1126.13	1125.56	-74.00	3151347.10
			13	1002.01	1243.89	-106.75	626139.39
			14	1118.73	1146.52	-75.78	3256817.50
:			15	1222.73	1175.65	2.89	299418.44
Parameters of the trend (a,b,s)			16	-608.37			
Corresponding confidence intervals:			1	2.8761	2.9766	2.8309	8720.9229
$\delta ?_k = \pm\sigma_k\, t_{(N-n-m),\alpha}$,?			2	2.9427	2.4554	5.5542	1250.7372
$\sigma_k = \sigma_v ? W_k$, $W_k = [\, a_{kk}\,]^{1/2}$;			3	9.3182	10.2536	10.0325	8292.7143
σ_v?= ?.70 mgal ,			4	4.0301	4.0115	4.8459	8711.2587
($t_{245,0.05}$ = 1.975			5	7.9660	4.0277	7.6470	6196.6537
$t_{245,0.01}$ = 2.604),			6	5.9582	6.0956	5.5464	8787.8704
a_{kk}, k=1, ..., n+m are the diagonal elements of the inverse of the matrix of the corresponding normal system of equations at the point of the minimum, arranged consequently by rows:			7	6.1025	5.6240	5.1369	8788.5534
			8	4.2195	4.1542	4.3197	8595.1153
			9	6.1198	5.4953	5.3163	8601.4711
			10	4.0099	4.3860	2.9492	8779.3444
			11	17.7712	32.5857	13.8057	8956.2595
			12	3.8290	4.4079	2.9395	8779.0838
			13	7.5273	6.4818	6.7098	8602.5008
			14	3.9914	4.2973	2.9540	8778.7052
			15	10.7273	6.4457	9.9376	8747.0338
Parameters of the trend (a,b,s):			16	11.5548			

Obviously, to obtain the masses in kilograms, the given quantities in the table must be multiplied by 1.5.10^2. For example, spheres with 0.360 gr/cm^3 density (i.e. — one approximately real density contrast) and 5 m radius, have a mass about 1.884.10^5 kg, more or less of the same order, as the masses of the main PS of the obtained solution. The real x and y co-ordinates — ξ_k and η_k respectively (in meters) can be obtained on the base of the following relations: $\xi_k = \xi_{k\,+\,649\,000\,[m]}$, $\eta_k = \eta_{k\,+\,4\,629\,000\,[m]}$.

— the functional

$$F(x) = \sum_{i=1}^{N} [y_i - f_i(x)]^2,$$

— the corresponding mean square deviation (MSD)

$$\sigma_v = [F(x)/(N-n-m)]^{1/2},$$

— the gradient of the functional

$$G(x) = \{\sum_{k=1}^{n} \left[\frac{\partial F(x)}{\partial x_k}\right]^2\}^{1/2},$$

— the coefficient of non-representativeness (Zhelev, 1970, 1972, 1974)

$$K_v = 100\left(F/E\right)^{1/2} \quad \%, \quad E = \sum y_i^2,$$

at the point of the minimum, etc.

Stability and errors in the solution

As is known, the problem concerning the exact evaluation of the errors in the solution in the non-linear case is not satisfactorily solved yet. But as in the close vicinity of the minimum, a linear representation is usually acceptable, the well-known formalism concerning statistical estimations of linear systems can be used in this and similar cases to study the stability of the solution. An approximation of the corresponding confidence intervals δx_k of the unknowns can be obtained by the following expression

$$\delta x_k = \pm \sigma_k t_{(N-n-m),\alpha}, \quad \sigma_k = \sigma_v \omega_k, \quad \omega_k = [a_{kk}]^{1/2},$$

where a_{kk}, k = 1, ..., n are the diagonal elements of the inverse matrix of the respective normal system of equations, and $t_{(N-n-m),\alpha}$ is the corresponding *t* score for the respective degrees of freedom (N-n-m) (m — the number of the trend parameters) and level of certainty α. Thus, we can have an approximate idea about the confidence intervals, suggesting an almost linear connection between the unknowns and the observations at the point of the minimum and its surroundings.

As can be easily seen from the table, almost all the PS and the trend are comparatively well determined — the corresponding errors in the solution are within acceptable limits.

Naturally, when this method is used, the question how to determine the optimal number of model parameters is essential. Although the optimization method used automatically eliminates the extra parameters of the model, in order to ease the optimization process however, the following additional method (Zhelev, 1990) can be employed for this purpose.

The problem can be solved for different numbers of elementary sources. The number *n* at which the corresponding MSD has a minimum, had to be chosen as an optimal one. It is not difficult to show, that if the number of the observations is large enough, there is a number of the ES at which this criterion has a minimum and this optimum coincides with the real number of the parameters of the source — the respective proof can be seen in (Zhelev, 1990). Obviously, the minimum value of the MSD thus obtained, must be approximately equal (or a little less) to (than) the corresponding mean square error in the observations, as it is its unbiased estimate (if a good representation is achieved) (see Zhelev, 1991, 1994). Thus, instead of searching for the minimum, we can look for that n for which the corresponding MSD coincides with (or is a little less than) the respective mean square error in the observations, when it is known of course.

Conclusion

The obtained results seem to be in better agreement with the karstic cavities filled with water or sediments supposed to be at different depths and the terrain collapses that have taken place in the last years in this region (and to a large degree clarify the available terrain's collapses in this region).

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Нови резултати от анализа на гравитационната аномалия при село Алакала де Ебро (Сарагоса, Испания) чрез множество от точкови източници

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Резюме. През последните години стават няколко пропадания на терена в района на село Алкала де Ебро (Сарагоса, Испания). Река Ебро минава близо до селото и действа на тази зона по един активен начин. Предполага се наличието на пещери, изпълнени с вода или седименти. Дълбочината на тези пещерите се оценява на около 12 до 20 m. Може да се каже, че няма други геоложки изследвания в района, които да осигурят допълнителна информация за тази цел. Съответната гравитационна аномалия от района е изследвана с модел от множество точкови източници (Zidarov, 1968; Zidarov and Zhelev, 1970; Zhelev, 1972; Zidarov, 1990; Zhelev at al., 1996).След предварителна полиноминална апроксимация за елеминиране на главната част на регионалния тренд (с цел олесняване на оптимизационния процес), съответната локална гравитационна аномалия (заедно с остатъка от тренда) се моделира с множество от 14 елементарни точкови източници и линеен тренд. Неизвестните параметри на предлагания модел се определят чрез оптимизация. Получените резултати добре се съгласуват с карстовите пещери пълни с вода или седименти, предполагани да съществуват на различна дълбочина в района и добре обясняват пропаданията на терена, ставащи в този район през последните години.