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Applicability of RETAS stochastic model to illustrate general seismicity in Izmit region, Turkey

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Приложимост на модела RETAS за описание на общата сеизмичност в региона на Измит, Турция

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Абстракт: Приложен е модел RETAS (ограничен епидемичен тип модел за афтершокова активност) за анализ на времевите характеристики на извадка от афтершоковата серия след земетресението с магнитуд M_w=7.4 от 17 август 1999 до гр. Измит, Турция. Показани са предимствата на модела, който включва в себе си и широко използваните модели ETAS (епидемичен тип модел за афтершокова активност) и MOF (модифицирана формула на Омори). Разгледани са афтершоци с магнитуд по-голям или равен на M_o=4.0. Идентифицираният най-добър модел е версия на RETAS, при която тригериращият магнитуд съвпада с долната магнитудна граница (RETAS=ETAS). Моделът добре описва реалните данни и спомага да се определи период на сравнителен недостиг на реални сеизмични събития преди най-силния афтершок (земетресението до гр. Дюзче). След това моделът е приложен за характеризиране на общата сеизмичност в изследвания регион за период от около девет години. Наблюдава се добро съвпадение на реалната и моделна кумулативни криви във времето. Това е важен резултат, като се има предвид, че моделните параметри са оценени върху данни за афтершоци за период само от 500 дни след главния трус. Една възможна интерпретация на тези резултати е евентуална връзка между моделните параметри и физични свойства на средата, така че оценките на параметрите само от афтершокови данни да важат и за описание на поведението на общата сеизмичност в региона.

Introduction

Numerous earthquake occurrence models have been developed to describe various aspects of seismic occurrence patterns, assuming that earthquakes are the realization of a stochastic point process. While generally strong earthquakes in a region are assumed to follow a stationary Poisson process, the clustering feature in earthquake occurrences has focused the researchers' attention for a long time. Depending on the tectonic structure of the regions, Mogi (1963) distinguishes three basic types of clusters: a) - a main shock followed by a number of aftershocks; b) increased seismicity rate (foreshocks), followed by a main shock and leading to a type (a) sequence; c) a gradual increase and decay of seismicity in time without a distinct main shock, known as swarm. The bigger part of studies in the literature aim to model the gradual decay of the aftershocks triggered by a strong earthquake in type (a) sequences. The most widely used model is the so-called Omori Law (OMORI, 1894), which UTSU (1961) transformed into the modified Omori formula (MOF). It is based on the assumption, that all the events in an aftershock sequence are triggered by the stress field change due to the main shock. All aftershocks are considered to be conditionally independent and follow a nonstationary Poisson process. The existence of sequences of more complex behavior with two or more main shocks or of several strong aftershocks required enrichment of the MOF with additional terms. Following such an approach OGATA (1988) introduced into the MOF the idea of self-similarity by assuming every aftershock of the sequence to be capable of generating secondary events. He formulated his idea through the Epidemic Type Aftershock-Sequence (ETAS) model. Besides the triggered seismicity the ETAS model may also describe the background activity, represented by a constant term added to the conditional intensity function. Between these two limit cases, the MOF model with only one parent-event and the ETAS model in which every event shares in the generation of the subsequent ones, there is a range of different versions of trigger models. Gospodinov and Rotondi (2006) offered the restricted epidemic type aftershock sequence (RETAS) model in which, as in Ogata (2001), triggering capabilities possess events with magnitudes larger than or equal to a threshold M_{th}. They developed the model motivated by Bath's law (Bath, 1973). By varying the triggering threshold, they examined the variants of the RETAS model which range from the Modified Omori Formula (MOF) to the ETAS model, including such models as limit cases. While RETAS is usually applied to model aftershock activity in a sequence, its mathematical formulation allows that it could be used to illustrate general seismicity, too. In this study we aim first at modeling the temporal behavior of the M7.4 Izmit earthquake sequence in 1999. Then we

try to utilize the RETAS model with the corresponding aftershock parameter estimates, to forecast statistically general seismicity in the region.

RETAS model formulation

The typical aftershock decay is represented by the modified Omori function (Omori, 1894)

$$n_a(t) = \frac{K}{(t+c)^p} \qquad (1)$$

where t is the time elapsed from the occurrence of the main shock, K is a parameter related to the magnitude of the main shock and to the cut-off magnitude M_0, p is a coefficient of attenuation and c is a constant. The conditional intensity function in the ETAS model is presented by

$$\lambda(t|H_t) = \mu + \sum_{t_i<t} \frac{K_0 e^{\alpha(M_i - M_0)}}{(t-t_i+c)^p} \qquad (2)$$

where μ (shocks/day) is the rate of background activity, the history H_t consists of the times t_j (days) and magnitudes m_j of all the events which occurred before t and the summation is taken over every jth aftershock with a magnitude stronger than the cut-off $m_j \geq M_0$ i.e. over all events in the sample. The coefficient α measures the magnitude efficiency of a shock in generating its aftershock activity and K_0 (shocks/day) measures the productivity of the aftershock activity during a short period right after the mainshock (cf. Utsu, 1970).
As mentioned above, the RETAS model considers that only aftershocks with magnitudes above certain threshold M_{th} are capable of generating secondary events, hence its conditional intensity function is provided by the formula

$$\lambda(t|H_t) = \mu + \sum_{\substack{t_i<t \\ M_i \geq M_{tr}}} \frac{K_0 e^{\alpha(M_i - M_0)}}{(t-t_i+c)^p} \qquad (3)$$

In fact the RETAS software (Gospodinov and Rotondi, 2006) verifies the model versions for all possible values of the threshold M_{th}, starting from the cut-off magnitude M_0 and up to the main shock magnitude. In that way RETAS considers the MOF and the ETAS models, too, as limit cases. It also considers all intermediate thresholds and this is one of the main advantages of the RETAS model compared to MOF and ETAS. The model version which best fits certain data is chosen on the basis of the Akaike criterion given by:

$$AIC = -2 \max \log L(Q;0,T) + 2k \qquad (4)$$

where k is the number of parameters of the model and $\log L$ is the logarithm of the likelihood function, given by

$$\log L(Q;0,T) = \sum_{i=1}^{N} \log \lambda_Q(t_i|\mathrm{H}_{t_i}) - \int_0^T \lambda_0(s|\mathrm{H}_s)ds \qquad (5)$$

In the above expression, N is the number of earthquakes of magnitude larger than or equal to M_0 which occur at times t_i; $i = 1; \ldots ; N$, in the interval under study $[0, T]$. The minimum value of AIC identifies the best model. After having chosen the best model among the RETAS versions with different M_{th}, we have evaluated its goodness of fit through the residual analysis. The integral

$$\tau = \Lambda(t) = \int_0^t \lambda(s)ds \qquad (6)$$

generates a 1-1 transformation of the time from t_i to τ_i so that the occurrence dates $t_1; t_2; \ldots ; t_N$ are transformed into $\tau_1; \tau_2; \ldots ; \tau_N$. It is known that the τ_i; $i = 1; \ldots ; N$, are distributed as the standard stationary Poisson process if $\lambda(s)$ is the intensity function of the process actually generating the data. The process on the

transformed axis is called residual process (OGATA, 1988) and provides a measure of the deviation of the data from the hypothesized model.

Data and obtained results

We analyze the seismicity in the seismic zone in which the Izmit M7.4 earthquake occurred on August 17, 1999. The Izmit earthquake is remarkable in being the latest in a series of 11 major (M>6.7) earthquakes in the last century that have broken more than a 1000 km length of the North Anatolian (NA) fault. This was not the first earthquake in this area. Records show that very similar events occurred in 1719 and 1894 on the North Anatolian Fault. This is an extremely active fault that accommodates the westward movement of Turkey as it is squeezed between the northward motion of the Arabian plate and the stationary Eurasian plate. The North Anatolian fault is a major, right-lateral, continental strike slip fault. The fault zone splays out into three strands at about 30.5 deg E (***Fig.1***). The northern strand crosses the Bay of Izmit and the Marmara Sea, and reappears in the Gulf of Saros. And this is this strand, along which the August 17 earthquake took place. The main shock was followed by an intense aftershock sequence, whose temporal properties come to be part of the topics under study in this paper.

Table 1. M_o – lower magnitude cut-off, M_{th} – threshold magnitude, *K, α, c, p,* - RETAS model parameters (see in text); *AIC* – Akaike criterion value

Best fit model	M_o	M_{th}	*K*	*α*	*c*	*p*	*AIC*
RETAS→ ETAS	4	4	0.009	2.436	0.039	1.21	-177.273

We decided to analyze aftershocks with lower magnitude cut-off $M_o \geq 4.0$ above which the sequence surely can be considered as complete. A catalog of N=127 events was compiled for a period of 500 days after the main shock. The results from the application of the RETAS software (see ***Table 1*** and ***Fig.2a***) revealed that aftershock rate decay is best fitted by a version of the model for which $M_{th} = M_o$. We use the parameters estimates to plot real (red circles) and model (thick blue line) cumulative numbers of events in time (see ***Fig.2b,c***).

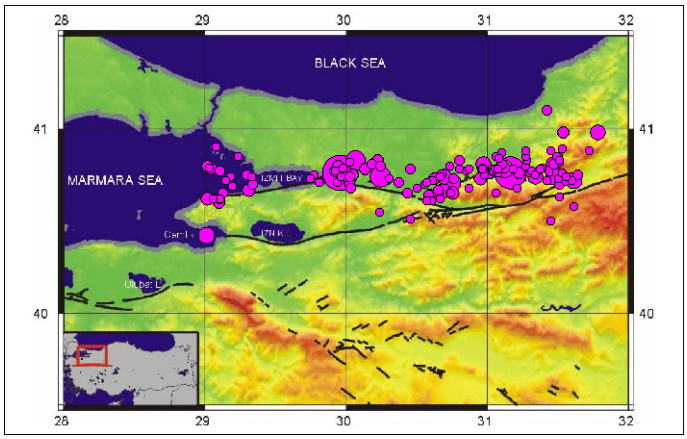


Fig.1 Epicenter map of earthquakes with M≥4.0 for the seismic zone of the Izmit M7.4 (17 August) and Duzche M6.6 (26 November) earthquakes of 1999. The considered time period is since the main M7.4 shock till November 2008

The cumulative curves on ***Fig.2b*** reveal that the identified model fits the data quite well; real data circles stay in between the error bounds for the entire period of 500 days. There is some deficiency of real aftershocks for the period before the strongest aftershock (Duzche earthquake), which could be regarded as a quiescence, but this speculation needs more thorough study. What we were most interested in was whether the model provided an adequate illustration of general seismicity after the examined period of 500 days.

On ***Fig.2c*** we plotted the real and model cumulative numbers up to October 2008 (nearly 9 years period). The model offers a very good fit of real seismicity for the whole period. The results from this comparison seem very interesting because the model parameters were estimated on data for only 500 days and the model illustrates general seismicity for nearly nine years.

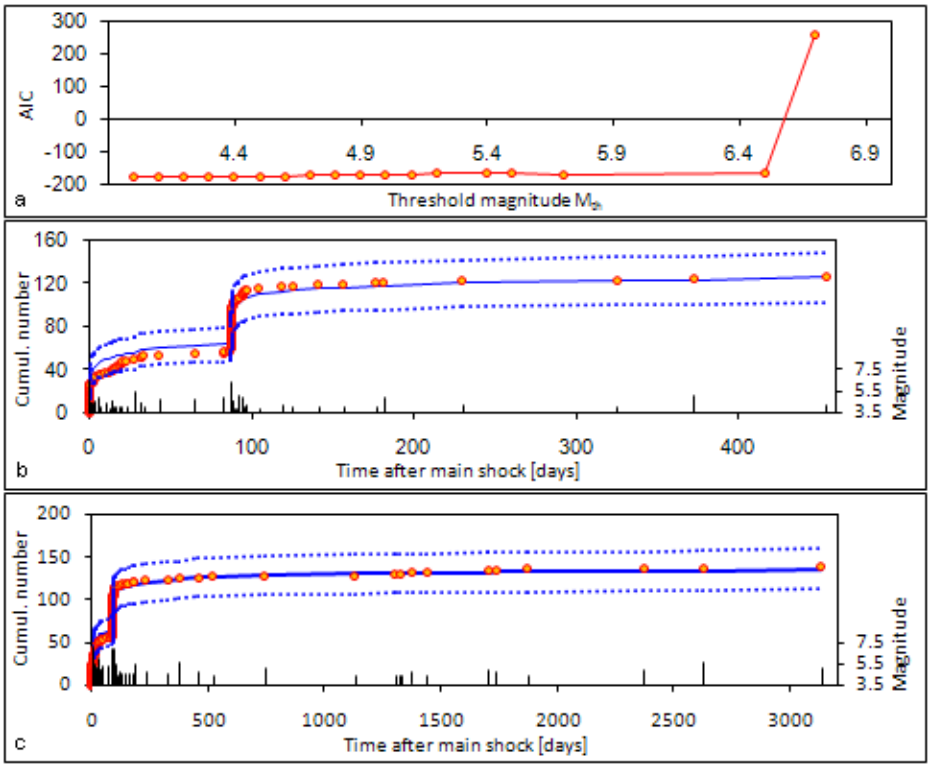


Fig.2 Results from the RETAS model application on catalog data of the *M7.4* earthquake zone (August 17, 1999) near Izmit; a) *AIC* values versus the magnitude threshold M_{th}, above which aftershock are capable to induce secondary aftershocks. The best fit model is for $M_{th}=M_o$ that is RETAS=ETAS; b) Real (circles) and model (thick blue line) cumulative numbers of aftershocks for a period of 500 days – model parameters were estimated on this data; c) Real (circles) and model (thick blue line) cumulative numbers for a period of nine years. Dotted lines stand for error bounds

We were tempted to believe that a possible interpretation of the latter results is an eventual relation between RETAS model parameter values and some physical properties of the seismogenic region.

Conclusions

RETAS model was applied first to analyze the aftershock data of the M7.4 Izmit earthquake. The study outlines the advantages of the model compared to ETAS and MOF. One can see the AIC values for all versions of the RETAS model and the smallest value identifies the best fit version. For this data it turned to be ETAS and it provides a good picture of aftershock decay, also presenting a possibility to recognize a period of quiescence before the strongest aftershock in the sequence. It is really interesting that the model offers a very good fit of real seismicity for a period of nine years after the strong M7.4 earthquake, although the model parameters were estimated on aftershock data for only 500 days. This infers a possible relation between RETAS model parameters and some physical properties related to the seismogenic process in the studied region.

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